LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – NOVEMBER 2012

# MT 5508/MT 5502 - LINEAR ALGEBRA

 Date : 08/11/2012 Dept. No. Max. : 100 Marks

 Time : 9:00 - 12:00

**PART – A**

**Answer ALL questions: (10 x 2 = 20 marks)**

1. Define a vector space over a field F.
2. Show that the vectors (1,1) and (-3, 2) in R2 are linearly independent over R, the field of real numbers.
3. Define homomorphism of a vector space into itself.
4. Define rank and nullity of a vector space homomorphism T: u→v.
5. Define an orthonormal set.
6. Normalise  in R3 relative to the standard inner product.
7. Define a skew symmetric matrix and give an example.
8. Show that  is orthogonal.
9. Show that  is unitary.
10. Define unitary linear transformation.

**PART – B**

**Answer any FIVE questions: (5 x 8 =40 marks)**

1. Prove that the intersection of two subspaces of a vector space v is a subspace of V.
2. If S and T are subsets of a vector space V over F, then prove that
3. S T implies that L(S) ≤ L(T)
4. L(L(S)) = L(S)
5. L(S U T) = L(S) + L(T).
6. Determine whether the vectors (1,3,2), (1, -7, -8) and (2, 1, -1) in R3 are linearly dependent on independent over R.
7. If V is a vector space of finite dimension and W is a subspace of V, then prove that

dim V/W = dim V – dim W.

1. For any two vectors u, v in V, Prove that .
2. If  and λ ∈ F, then prove that λ is an eigen value of T it and only if [λ I – T] is singular.
3. Show that any square matrix can be expressed as the sum of a symmetric matrix and a skew symmetric matrix.
4. For what values of T, the system of equations  over the rational field is consistent?

**PART – C**

**Answer any TWO questions: (2 x 20 = 40 marks)**

1. a) Prove that the vector space V over F is a direct sum of two of its subspaces W1 and W2

 if and only if V = W1 + W2 and W1  W2 = {0}.

b) If V is a vector space of finite dimension and is the direct sum of its subspaces U and

 W, than prove that dim V = dim U + dim W. (10 + 10)

 20. If U and V are vector spaces of dimension m and n respectively over F, then prove that the

 vector space Hom (U, V) is of dimension mn.

 21. Apply the Gram – Schmidt orthonormalization process to obtain an orthonormal basis for

 the subspace of R4 generated by the vectors (1, 1, 0, 1) , (1, -2, 0, 0) and (1, 0, -1, 2).

 22. a) Prove that T∈A(V) is singular if and only it there exists an element v ≠ 0 in V such that

 T(v) = 0.

 b) Prove that the linear transformation T on V is unitary of and only if it takes an

 orthonormal basis of V onto an orthonormal basis of V. (10 +10)

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